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First Order Noise Shaping Local-Oscillator Based Time-to-Digital Converter

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***Abstract*— This paper presents a First Order Noise Shaping Local-Oscillator Based Time-to-Digital Converter (LO TDC). The architecture and governing equations of the LO TDC are described. In order to show the effect of noise shaping on the resolution of the TDC, the system "LO TDC plus moving average filter" is introduced. An equation to predict the resolution of the system "LO TDC plus filter" is given. Then, the Matlab model of the system "LO TDC plus filter" is illustrated briefly, and some example of simulated input-output characteristics are shown. Afterwards, the implementation of the LO TDC on an FPGA is described. A comparison between the predicted, simulated and measured values of the resolution of the system "LO TDC plus filter" is reported. Finally, we show spectra of the output signals of the LO TDC from experiments and simulations.**

I. Introduction

Precise measurements of time intervals between two events are fundamental in many applications that require time of flight measurements such as Positron Emission Tomography imaging [1] and radar systems [2], or clock related measurements, such as jitter measurements [3] and All-Digital-PLLs (ADPLLs) [4 – 6]. A Time-to-Digital Converter (TDC) measures an unknown time interval by counting how many reference time intervals it includes. The duration of the input time interval is typically determined by the edges of two digital signals, as shown in Fig. 1.

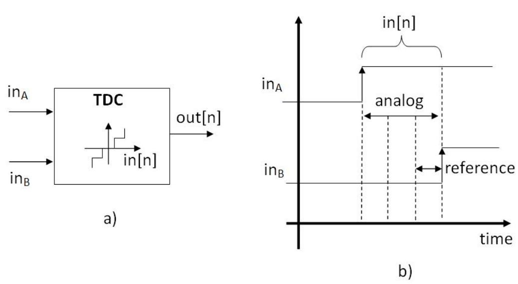

Fig. 1. *Concept of a Time-to-Digital Converter.*

There are two main kinds of reference time intervals: (i) the propagation delay of delay elements [7] and (ii) the oscillation period of oscillators [8]. Due to its finite resolution, a TDC introduces quantization noise on the output. Several techniques have been proposed in the literature to augment the resolution of TDCs and to reduce the quantization noise. These include Vernier architectures [9], interpolation techniques [8] and first order noise shaping TDCs [10].

In noise shaping TDCs, the resolution and signal to noise ratio at low frequencies are improved by pushing the power of the quantization noise to high frequencies, where it can later be removed by low-pass filtering. This feature makes noise shaping TDCs particularly interesting in ADPLLs, where the loop filter can be used to suppress the noise at high frequencies.

II. Architecture of the first order noise shaping LO TDC

First order noise shaping of the quantization error is implemented when the measurement n includes the quantization error of the previous measurement $(n-1)$. In [10] and [11], a Gated Ring Oscillator based TDC (GRO TDC) is used to implement the first order noise shaping. In the GRO TDC, the quantization error of every measurement is associated with the output charge of every stage of the GRO. The GRO is enabled only during the time intervals that have to be measured, such as those that are associated with the phase error in an ADPLL. First order noise shaping is achieved by preserving the output charge of the GRO between two consecutive measurements. In fact, if at the beginning of the current measurement the value of the output charge of the GRO is the same of that at the end of the previous measurement, the quantization error of the previous measurement is included in the current one. The redistribution of the output charge and the transient of the GRO are the main implementation issues of the GRO TDC architecture.

In [12], we proposed the first order noise shaping Local Oscillator based TDC as an alternative to the GRO TDC. In the LO TDC, the oscillator is never disabled and the time intervals to be measured are consecutive. A sampler selects the data associated with the interesting time intervals and discards the others. In the LO TDC, the value of the output charge of the LO at the beginning of the measurement n is equal to the value of the output charge of the end of the previous measurement $n-1$, by continuity. Therefore, in the LO TDC, first order noise shaping is implemented without any issue related to the charge redistribution or transient of the oscillator.

The architecture of the LO TDC that is proposed in [12] is illustrated in Fig. 2a. In this work we consider the simplified

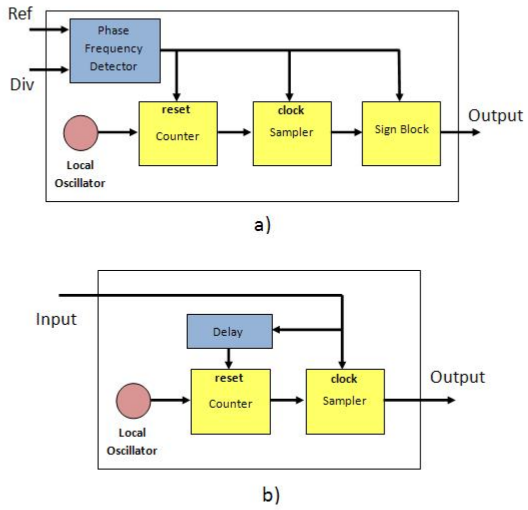

Fig. 2. *a) Block diagram of the LO TDC presented in [12]; b) block diagram of the LO TDC adopted in this work.*

LO TDC architecture shown in Fig. 2b. The difference between the two architectures in Fig. 2 is that in the simplified LO TDC the time interval that has to measured is defined by the consecutive rising and falling edges of a single signal, while in the LO TDC the edges of two signals are used. Despite this difference, first order noise shaping in the simplified LO TDC is implemented in the same way of the LO TDC, as can be seen in Fig. 3. In the next sections we will use the term LO TDC to refer to the simplified LO TDC.

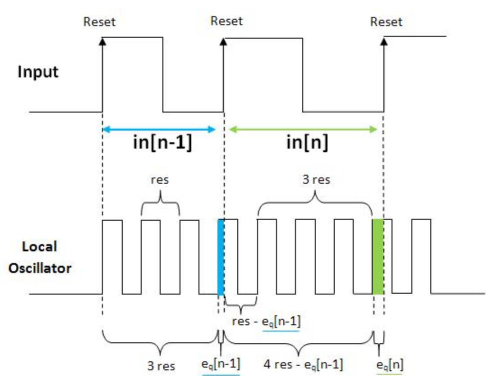

Fig. 3. *Example of how the first order noise shaping of the quantization error is implemented in the simplified LO TDC.*

III. Theoretical Performance

In this section we report the governing equations of zeroth and first order noise shaping TDCs. We also give an equation to predict the improvements in terms of resolution related to first order noise shaping.

The output of a zeroth order noise shaping TDC, namely a simple quantizer, is given by:

$$out[n] = res \cdot \text{floor}\left(\frac{in[n]}{res}\right) = in[n] + e_q[n], \tag{1}$$

where in is the time interval to measure, res is the resolution, the $\text{floor}(\cdot)$ operator rounds to the closest integer less than the argument, and $e_q[n]$ is the quantization error at step n. From Eq. (1) it is possible to obtain the example of the input-output characteristics of a zeroth order noise shaping TDC that is shown in Fig. 4a.

The quantization error of a zeroth order noise shaping TDC is:

$$e_q[n] = res \cdot \text{floor}\left(\frac{in[n]}{res}\right) - in[n], \tag{2}$$

By contrast, the equation which relates the input and output of a first order noise shaping TDC is:

$$out[n] = res \cdot \text{floor}\left(\frac{in[n] - e_q[n-1]}{res}\right). \tag{3}$$

As can be seen from Eq. (3) $out[n]$ is an integer multiple of res. By contrast, $in[n]$ and $e_q[n]$ can be equal to any real number. The quantization error of a first order noise shaping TDC is:

$$e_q[n] = res \cdot \text{floor}\left(\frac{in[n] - e_q[n-1]}{res}\right) - in[n] + e_q[n-1]. \tag{4}$$

By substituting (4) into (3), we obtain:

$$out[n] = in[n] - e_q[n-1] + e_q[n]. \tag{5}$$

In the z-domain, Eq. (5) can be written as:

$$Out(z) = In(z) + E_q(z)(1 - z^{-1}). \tag{6}$$

Equation (6) shows that the quantization error is first order shaped.

As can be derived from Eqs. (3) and (4), when the input of a first order noise shaping TDC with resolution res is a constant $coef$, the output is a sequence of samples equal to the two integer multiples of res closest to $coef$. Figure 4b shows an example of the output signal of a first order noise shaping TDC when $ref = 1$ and $coef = 0.0479$, namely a value much smaller than res.

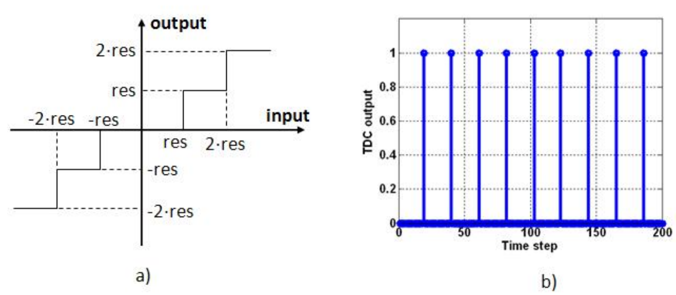

Fig. 4. *a) Example of input-output characteristics of a zeroth order noise shaping TDC with resolution res; b) example of the output signal of a first order noise shaping TDC with $res = 1$ when the input is a constant much smaller than the res.*

It can be proven by calculating the average value of the TDC output from Eqs. (3) and (4) and verified by means of simulations that the average value of the output signal of the TDC is proportional to the input signal. A digital moving average filter with window size equal to N_{avg} samples can be used to process the output of the first order noise shaping TDC to obtain a digital measure of the input signal. The output of the filter is given by:

$$out_{filt}[n] = \frac{1}{N_{avg}} \sum_{i=n-N_{avg}+1}^{n} in_{filt}[i], \tag{7}$$

In particular, it can be proven from Eqs. (3), (4) and (7) that the resolution of the whole system "first order noise shaping TDC plus filter" is smaller than or equal to:

$$res' = res/N_{avg}, \tag{8}$$

where res is the resolution of the first order noise shaping TDC and N_{avg} is the number of samples used by the moving average filter. We will show that a first order noise shaping TDC with a moving average filter is equivalent to a zeroth order noise shaping TDC with resolution smaller than or equal to res', provided that the noise components at high frequencies are removed by the loop filter, as illustrated in Fig. 5.

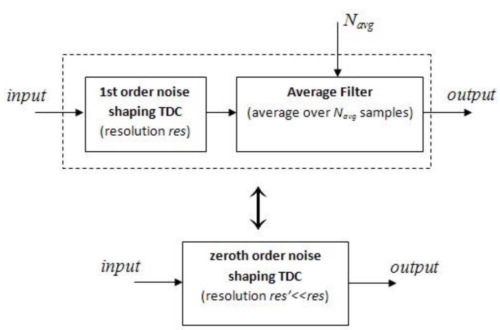

Fig. 5. *First order noise shaping TDC with a moving average filter.*

IV. Simulated Performance

Figure 6 shows the block diagram of the model of the LO-TDC in Fig. 2b that we implemented in Matlab.

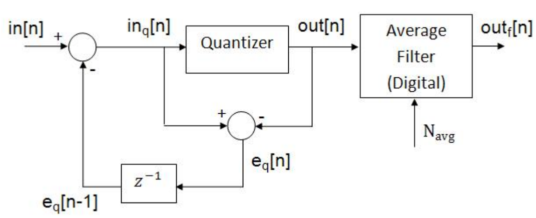

Fig. 6. *Matlab model of the first order noise shaping LO TDC with a moving average filter.*

Equations (3) and (4) can be derived from the block diagram in Fig. 6. We have verified by simulation that the Matlab model of the LO-TDC is equivalent to the verilog-AMS exact TDC models proposed in [12]. Figure 7 shows the input-output characteristics of the system "LO TDC plus filter" obtained with Matlab simulations of the system "LO TDC plus filter", when $res = 1$ and $N_{avg} = 5, 10$ and 100, respectively. The size of the quantization step of the characteristics decreases when N_{avg} increases.

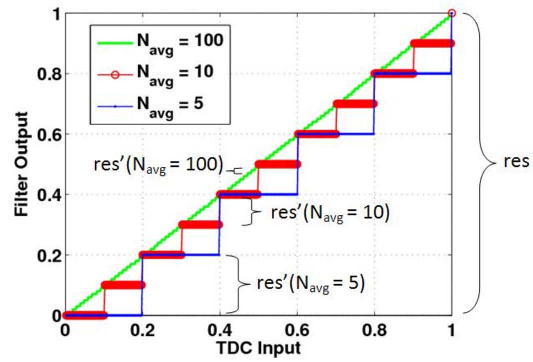

Fig. 7. *Simulated input-output characteristics of the system "LO TDC plus filter" with res = 1 and N_{avg} = 5, 10, 100.*

V. FPGA implementation

We realized the LO TDC on a Virtex-5 LX Xilinx Protoboard FF676. Figure 8 shows the preliminary experimental setup that we used to test the system "LO TDC plus filter". Since the moving average filter is not yet implemented on the FPGA, the output of the LO TDC is processed with a moving average filter in Matlab. An Agilent pulse generator 81130A provides the clock to the board at 2 MHz. The on-board PLLs, labeled $PLL1$ and $PLL2$ in Fig. 8, are used to generated an input signal at 40 MHz and a reference signal at 1.91667 MHz, respectively. The period of the output of $PLL1$, which is equal to 25 ns, is the time interval that has to be measured. The period of $PLL2$, which is equal to about 500 ns, is the resolution res of the LO TDC. It is important to notice that the reference time interval is about 20 times greater than the period of the signal that we want to measure. An on-board Integrated Logic Analyzer (ILA) samples the output of the LO TDC. Finally, Xilinx ChipScope Analyzer 11.1 is used to access to the data of the ILA.

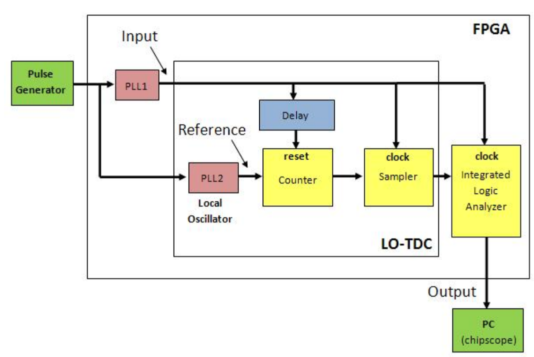

Fig. 8. *Block diagram of the experimental setup and circuit implemented on FPGA.*

Figure 9 shows a comparison between the simulated, measured and predicted (with Eq. (8)) normalized values of the resolution of the system as a function of N_{avg}. The experimental data are very close to the simulation results. As can be seen from Fig. 9 the resolution of the system "LO TDC plus filter" implemented on FPGA has been increased from about 500 ns to about 4 ns by increasing N_{avg} from 1 to 100. Notice that the resolution of "LO TDC plus filter" has been increased without reducing the size of the reference time interval.

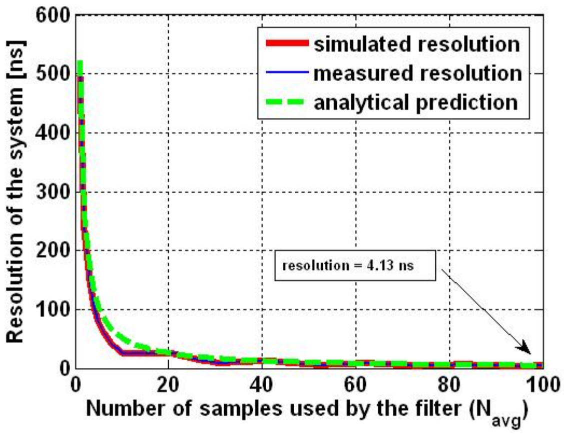

Fig. 9. *Resolution of the system (res'/res) as a function of* N_{avg} *from predictions, simulations and experiments.*

The main drawback of first order noise shaping TDCs is the presence of tones and noise at high frequency, which are not present in the output spectra of zeroth order noise shaping TDCs. In order to suppress the tones by means of the loop filter, it is important to be able to predict the positions and amplitudes of the tones. Our Matlab model of the LO TDC can be used for this purpose efficiently. In fact, the execution times of the Matlab simulations are a few seconds. Figure 10 shows a comparison between simulated and measured power spectral densities of the output signal of the LO TDC when the frequencies of PLL1 and PLL2 are 40 MHz and 1.91667 MHz, respectively. The simulation results are very close to the measurement. The main difference is due to the noise floor, that is not present in the simulation because no noise has been added to the constant input of the Matlab model of the LO TDC.

VI. Conclusions

In this paper we have presented a First Order Noise Shaping Local-Oscillator Based Time-to-Digital Converter (LO TDC). The architecture of the system "LO TDC plus moving average filter" is introduced. The matlab model and FPGA implementation of the LO TDC are described. We give an analytical equation to predict the value of the resolution of the system "LO TDC plus filter". We show that the resolution of the system "LO TDC plus filter" is reduced by increasing the number of samples used by the moving average filter. The experimental results exhibit good agreement with simulations and analytical equations.

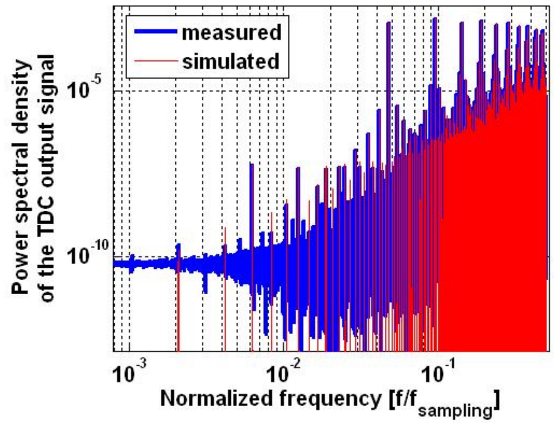

Fig. 10. *Spectrum of the output signal of the LO TDC from simulations and experiments.*

VII. Acknowledgements

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